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# SD-Divisibility and Some Results on SD-Divisor Labeling of Graphs

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## ABSTRACT

Let G with bijection f :V→{1, 2,..., n}, linked to S = f(x) + f(y) and D = |f(x) − f(y)| with each edge xy in E. The labeling f makes on edge labeling f* : E → {0, 1} like for any xy in E, f*(xy) = 1 if D | S and f*(xy) = 0 if D ∤ S. We mean f is an SD-divisor labeling if f*(xy) = 1 for all xy in E. Further, G is SD-divisor if it have SD-divisor labeling. We mean f is an SD-divisor cordial labeling if $| e_{f*}(0) - e_{f*}(1)| \leq 1$. Further, G is SD-divisor cordial if it have SD-divisor cordial labeling. In this article, we define SD-divisibility and SD-divisor pair of numbers and establish some of its properties. We also proved some standard graphs are not SD-divisor.

**Keywords -** Divisor cordial labeling, SD-divisor labeling, SD-divisor graph

## 1. INTRODUCTION

In this article, graphs and number theory terms we mention to [4] and [2] respectively. Also divisor cordial labeling and SD-prime labeling concepts we refer to [10] and [6] respectively. Here we define SD-divisibility and SD-divisor pair of numbers and establish some of its properties. Also we proved some standard graphs like star, complete, complete bipartite and wheel graphs are not SD-divisor.

## 2. SD-DIVISIBILITY AND ITS PROPERTIES

First we define SD-divisibility of two positive integers.

Definition 2.1 Let a and b be the two distinct positive integers, we say that a SD-divides b if |a − b| | (a + b). It is indicated by $a \mid_{SD} b$. If a does not SD-divide b, then it is indicated by $a \nmid_{SD} b$.

Example 2.2 $3 \mid_{SD} 5$.

Example 2.3 $2 \nmid_{SD} 10$.

Note:

1. From the examples 2.2 and 2.3, divisibility and SD-divisibility are different concepts.
2. By the definition 2.1, SD-divisibility is not reflexive.
3. From the definition 2.1, $a \mid_{SD} b$ gives |a − b| | (a + b). Its gives |b − a| | (b + a). Clearly, $b \mid_{SD} a$. Thus, SD-divisibility is symmetric.
4. SD-divisibility is not transitive.
5. SD-divisibility is not an equivalence relation.

Example 2.4 $1 \mid_{SD} 3$ and $3 \mid_{SD} 5$ but $1 \nmid_{SD} 5$.

Observation 2.5 It is known that if k and k+2 are two consecutive odd integers, then k ∤ k + 2 for k ≥ 3. If k and k + 2 are two consecutive even integers, then k ∤ k + 2 for k ≥ 4.

Result 2.6 1 SD-divides only to the integers 2 and 3.
Proof: If a = 1 and b > 1 are any positive integer.
If $1 \mid_{SD} b$, then (b−1) | (b+1).
This means that two consecutive odd integers or two consecutive even integers divide.
This is possible only if b = 2 and 3.

Result 2.7 2 SD-divides only to the integers 1, 3, 4 and 6.
Proof: If a = 2 and b are any positive integer.
If $2 \mid_{SD} b$, then |b − 2| | (b + 2).
This is possible only if b = 1, 3, 4 and 6.

Result 2.8 3 SD-divides only to the integers 1, 2, 4, 5, 6 and 9.
Proof: If a = 3 and b are any positive integer.
If $3 \mid_{SD} b$, then |b − 3| | (b + 3).
This is possible only if b = 1, 2, 4, 5, 6 and 9.

Observation 2.9 Let a ≥ 3 be the any positive integer. Then a − 1, a − 2, a + 1, a + 2, 2a and 3a are SD-divisible by a.

Result 2.10 If a and b are the two consecutive odd integers, then $a \mid_{SD} b$.
Proof: Let a = 2k + 1, b = 2k + 3 for k ≥ 0.
Then |a − b| = 2 and a + b = 4(k + 1).
Clearly 2 | 4(k + 1). Then $a \mid_{SD} b$.

Result 2.11 If a and b are the two consecutive even integers, then $a \mid_{SD} b$.
Proof: Let a = 2k + 2, b = 2k + 4 for k ≥ 0.
Then |a − b| = 2 and a + b = 2(2k + 3).
Clearly 2 | 2(2k + 3). Then $a \mid_{SD} b$.

## 3. SD-DIVISOR PAIR

In this section, we define SD-divisor pair of integers and establish some results.

Definition 3.1 If a and b are the two distinct positive integers. If $a \,|_{SD}\, b$, then we say that (a, b) is called SD-divisor pair.

Example 3.2 For $k \geq 1$, $(k, k + 1)$ is SD-divisor pair.

Note: If $l \geq 1$ is any positive integer, then $(lk, l(k + 1))$ is SD-divisor pair.

Result 3.3 If (a, b) is SD-divisor pair, then (ka, kb) is SD-divisor pair for $k \geq 1$.
Proof: Let a and b be the SD-divisor pair. Without loss of generality, we take $b < a$.
Then $a \,|_{SD}\, b$ gives $(a - b) \mid (a + b)$
$\Rightarrow k(a - b) \mid k(a + b)$ for $k \geq 1$
$\Rightarrow (ka - kb) \mid (ka + kb)$.
$\Rightarrow ka \,|_{SD}\, kb$.

Result 3.4 Let $k \geq 2$. Then $(k + 1, k - 1)$ is SD-divisor pair.
Proof: If $a = k + 1$ and $b = k - 1$ for $k \geq 2$.
Next $|a - b| = 2$, $a + b = 2k$.
Clearly $2 \mid 2k$.
Thus, $(k + 1, k - 1)$ is SD-divisor pair for $k \geq 2$.

Result 3.5 Let $k \geq 0$. Then $(2^k, 2^{k+1})$ is SD-divisor pair.
Proof: Let $a = 2^k$, $b = 2^{k+1}$ for $k \geq 0$.
Next $|a - b| = 2^k$ and $a + b = 3 \cdot 2^k$.
Clearly $|a - b| \mid (a + b)$.
Thus, $(2^k, 2^{k+1})$ is SD-divisor pair for $k \geq 0$.

Result 3.6 Let $k \geq 0$. Then $(3^k, 3^{k+1})$ is SD-divisor pair.
Proof: Let $a = 3^k$ and $b = 3^{k+1}$ for $k \geq 0$.
Then $|a - b| = |3^k - 3^{k+1}| = 2 \cdot 3^k$
and $a + b = 3^k + 3^{k+1} = 4 \cdot 3^k$.
Clearly $|a - b| \mid (a + b)$.
Thus, $(3^k, 3^{k+1})$ is SD-divisor pair for $k \geq 0$.

Result 3.7 Let $l \geq 4$ and $k \geq 0$. Then $(l^k, l^{k+1})$ is not SD-divisor pair.
Proof: Let $a = l^k$ and $b = l^{k+1}$ for $l \geq 4$, $k \geq 0$.
Then $|a - b| = |l^k - l^{k+1}| = l^k(l - 1)$
and $a + b = l^k + l^{k+1} = l^k(l + 1)$.
Clearly $l - 1 \nmid l + 1$ for $l \geq 4$.
Thus, $(l^k, l^{k+1})$ is not SD-divisor pair for $l \geq 4$, $k \geq 0$.

Definition 3.8 Let S be a set of any distinct positive integers. Then S is said to be SD-divisor set if every pair of integers in S is SD-divisor.

Notations 3.9 $[n] = \{1, 2, ..., n\}$

Example 3.10 [2] and [3] are SD-divisor sets.

## 4. GRAPHS WHICH ARE NOT SD-DIVISOR

In this section, we prove some standard graphs are not SD-divisor.

Definition 4.1 [5] A bijection $f : V \rightarrow \{1, 2, ..., n\}$ makes an edge labeling $f^* : E \rightarrow \{0, 1\}$ like for any edge xy in E, $f^*(xy) = 1$ if $f(x) \,|_{SD}\, f(y)$ and $f^*(xy) = 0$ if $f(x) \nmid_{SD} f(y)$. We mean f is an SD-divisor labeling if $f^*(xy) = 1$ for all xy in E. Further, G admits SD-divisor labeling is called SD-divisor graph.

Next, we will investigate whether the star graph $K_{1,n}$ is SD-divisor or not.

Clearly $K_{1,1}$ and $K_{1,2}$ are SD-divisor.

Fig. 1 shows $K_{1,3}$, $K_{1,4}$ and $K_{1,5}$ are SD-divisor.

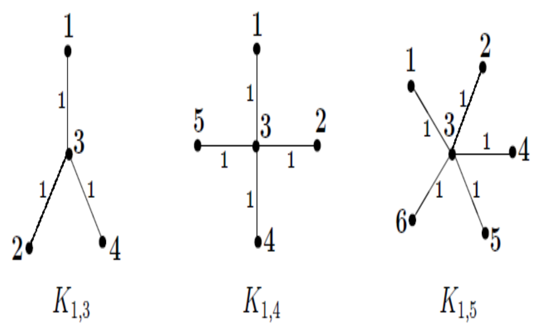


Figure 1. Star Graphs

Theorem 4.3 For $n \geq 6$, the star graph $K_{1,n}$ is not SD-divisor.
Proof: Consider the set $\{1, 2, ..., n + 1\}$, $n \geq 6$.
Let v be the central vertex of $K_{1,n}$ ($n \geq 6$).
If we label 1 to v and other numbers to the end vertices of $K_{1,n}$, then it follows from the result 2.6, 1 does not SD-divide 4, 5, 6, 7, 8, ..., n + 1.
If we label 2 to v and other numbers to the end vertices of $K_{1,n}$, then it follows from the result 2.7, 2 does not SD-divide 5, 7, 8, 9, 10, ..., n + 1.
If we label 3 to v and other numbers to the end vertices of $K_{1,n}$, then it follows from the result 2.8, 3 does not SD-divide 7, 8, 10, 11, 12, ..., n + 1.
Suppose, we label $n \geq 4$ to v.
Since any one of the end vertex has the label 1, then it follows from the result 2.6, 1 does not SD-divide to the label of v.
Thus, $K_{1,n}$ is not SD-divisor for $n \geq 6$.

Theorem 4.4 If $\delta(G) \geq 3$, then G is not SD-divisor.
Proof: Suppose G is SD-divisor.
Let v be the vertex of degree $\delta(G) \geq 3$, which is labeled with 1.
Then, any one of the $\delta$ adjacent vertices of v must have the labels other than 2 and 3, say w.

From the result 2.6, it follows that 1 does not SD-divide the label of w.
This is contradiction to G is SD-divisor.

Next, we discuss the SD-divisibility of complete graphs. Clearly, the complete graphs $K_1$, $K_2$ and $K_3$ are SD-divisor. Now, we will prove $K_n$ is not SD-divisor for $n \geq 4$.

Corollary 4.5 For $n \geq 4$, the complete graph $K_n$ is not SD-divisor.
Proof: Since $\delta(K_n) \geq 3$ for $n \geq 4$, the result follows from theorem 4.4.

Next, we discuss the SD-divisibility of complete bipartite graphs. Clearly, $K_{1,1}$, $K_{1,2}$ and $K_{2,2}$ are SD-divisor.

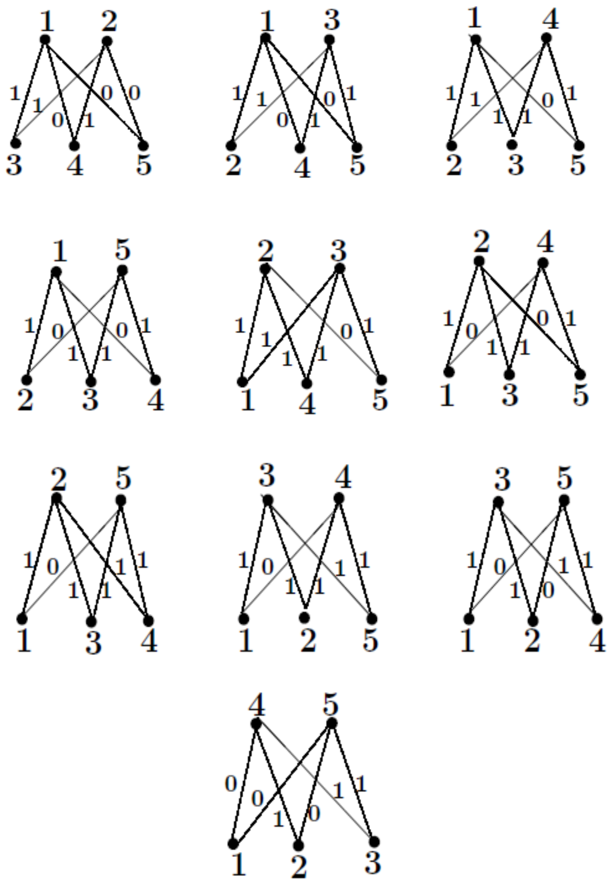


Figure 2. shows the labeling patterns $K_{2,3}$ is not SD-divisor. *Complete Bipartite Graph $K_{2,3}$*

Similarly, $K_{3,2}$ is not SD-divisor.

Corollary 4.6 For $m, n \geq 3$, the complete bipartite graph $K_{m,n}$ is not SD-divisor.
Proof: Since $\delta(K_{m,n}) \geq 3$ for $m, n \geq 3$, the result follows from theorem 4.4.

Corollary 4.7 The wheel graph $W_{n+1}(n \geq 3)$ is not SD-divisor.

Corollary 4.8 Petersen graph is not SD-divisor.

Corollary 4.9 Let $G_1$ and $G_2$ be SD-divisor graphs with $m \geq 2$ and $n \geq 2$ vertices respectively, then $G_1 + G_2$ is not SD-divisor.
Proof: Since $m \geq 2$ and $n \geq 2$, $\delta(G_1 + G_2) \geq 3$. Then the result follows from theorem 4.4.

## 5. CONCLUSION

In this article, we have introduced SD-divisibility and SD-divisor pair of numbers and establish some of its properties. We also proved some standard graphs are not SD-divisor. We have to find SD-divisor labeling of various type graphs. Also trying to find some applications using SD-divisor labeling in network problems.

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